

COMPOUND PENDULUM (Physical Pendulum)

Course: MJC-2, Physics

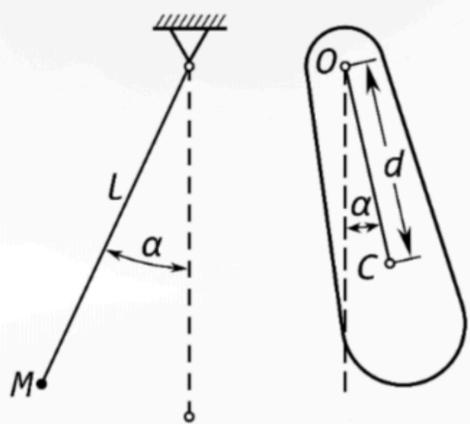
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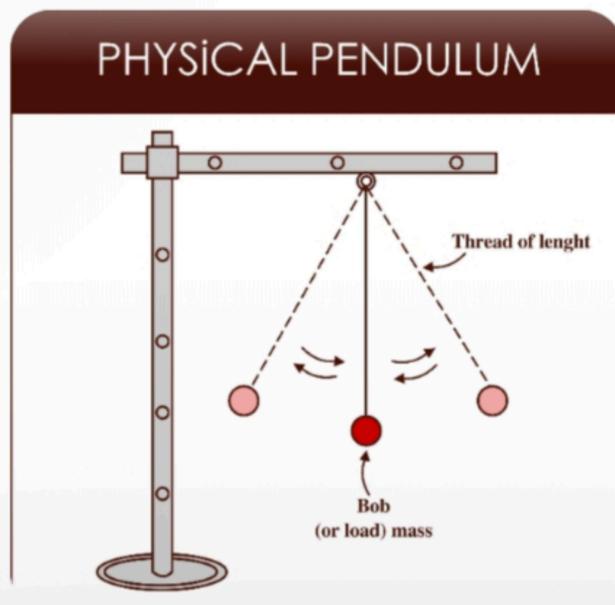
Prepared by: Usha Kumari

Division: Physics

Department: Physics



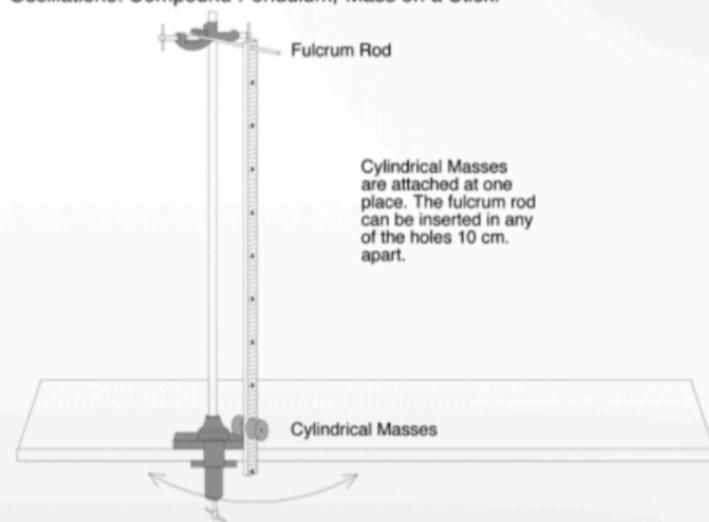
Mathematical and physical pendulums



SIMPLE HARMONIC MOTION.

B+10+3

Oscillations: Compound Pendulum, -Mass on a Stick.



1. Introduction

A **compound pendulum** (also called a **physical pendulum**) is a rigid body that is free to oscillate about a fixed horizontal axis under the influence of gravity. Unlike a simple pendulum, the mass of a compound pendulum is not concentrated at a single point but distributed throughout the body.

2. Definition

A compound pendulum is defined as **any rigid body capable of oscillating in a vertical plane about a fixed horizontal axis passing through a point other than its centre of mass.**

3. Theory of Compound Pendulum

Consider a rigid body of mass **M** suspended from a point **O**, at a distance **l** from its centre of mass **G**.

- Moment of inertia about centre of mass = **I_G**
- Moment of inertia about point of suspension (O):

$$I_O = I_G + Ml^2$$

4. Time Period of a Compound Pendulum

When the pendulum is displaced slightly and released, restoring torque acts on it.

Restoring torque:

$$\tau = -Mgl \sin \theta \approx -Mgl\theta$$

Using rotational equation of motion:

$$I_O \frac{d^2\theta}{dt^2} = -Mgl\theta$$

$$\frac{d^2\theta}{dt^2} + \frac{Mgl}{I_O} \theta = 0$$

This represents **simple harmonic motion (SHM)**.

Hence, time period:

$$T = 2\pi \sqrt{\frac{I_O}{Mgl}}$$

Substituting $I_O = I_G + Ml^2$:

$$T = 2\pi \sqrt{\frac{I_G + Ml^2}{Mgl}}$$

5. Equivalent Length of Compound Pendulum

Comparing with time period of a simple pendulum:

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Equivalent length **L** is:

$$L = \frac{I_G + Ml^2}{Ml}$$

6. Centre of Oscillation

The point at a distance L from the point of suspension is called the **centre of oscillation**.

Important result:

If the pendulum is suspended from the centre of oscillation, the time period remains the same.

7. Interchangeability of Centres

- Centre of suspension and centre of oscillation are **interchangeable**
- Time period remains unchanged if suspension point is shifted between these two points

8. Minimum Time Period

The time period is minimum when:

$$l = k \quad (\text{radius of gyration})$$

Minimum time period:

9. Comparison: Simple vs Compound Pendulum

Simple Pendulum

Compound Pendulum

Mass at a point

Mass distributed

Ideal system

Real system

Length fixed

Effective length

Easier theory

More practical

10. Advantages of Compound Pendulum

- More accurate than simple pendulum
 - Used to determine **g** precisely
 - Practical realization of oscillatory motion
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11. Applications

- Determination of acceleration due to gravity (**g**)
 - Used in clocks and oscillatory systems
 - Study of rotational dynamics
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12. Conclusion

A compound pendulum provides a realistic model of oscillatory motion and plays a vital role in understanding rotational mechanics. Its study bridges the gap between ideal systems and real physical bodies.
